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Tracing the string: BMN correspondence at finite J^2/N

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Abstract: Employing the string bit formalism of [18], we identify the basis transformation that relates BMN operators in $\mathcal{N} = 4$ gauge theory to string states in the dual string field theory at finite $g_2 = J^2/N$. In this basis, the supercharge truncates at linear order in g_2, and the mixing amplitude between 1 and 2-string states precisely matches with the (corrected) answer of [5] for the 3-string amplitude in light-cone string field theory. Supersymmetry then predicts the order g_2^2 contact term in the string bit hamiltonian. The resulting leading order mass renormalization of string states agrees with the recently computed shift in conformal dimension of BMN operators in the gauge theory.

Keywords: String Field Theory, Penrose limit and pp-wave background.

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1. Introduction and philosophy

The BMN correspondence [1] equates type IIB string theory on a plane wave background with a certain limit of $\mathcal{N}=4$ gauge theory at large R-charge J, where N is taken to infinity while the quantities

$$\lambda' = \frac{g_{\rm YM}^2 N}{J^2}, \qquad g_2 = \frac{J^2}{N} \tag{1.1}$$

are held fixed. The proposal is based on a natural identification between the basis of string theory states and the basis of gauge theory operators, and between the light-cone string hamiltonian P^- and the generator Δ of conformal transformation in the gauge theory via[1]

$$\frac{2}{\mu} P^- = \Delta - J\,. \tag{1.2}$$

BMN argued, and it was subsequently confirmed to all orders in λ' [2, 3], that this identification holds at the level of free string theory ($g_2 = 0$).

This beautiful proposal equates two operators which act on completely different spaces: the light-cone hamiltonian P^- acts on the Hilbert space of string field theory, and allows for the splitting and joining of strings, while $H \equiv \Delta - J$ acts on the operators of the field theory, and in general mixes single-trace operators with double- and higher-trace operators. Light-cone string field theory in the plane wave background has been constructed in [4, 5]. On the field theory side, a number of impressive papers [7]–[14] have pushed the calculations to higher order in g_2 with the aim of showing that (1.2) continues to hold, thereby providing an equality between a perturbative, interacting string theory and perturbative $\mathcal{N}=4$ gauge theory. It is clear, however, that at finite g_2 the natural identification between single string states and single trace operators breaks down. For example, 1-string states are orthogonal to 2-string states for all g_2, but single-trace operators and double-trace operators are not. This raises the question how to formulate the BMN correspondence in the interacting string theory.

[1] The parameter μ can be introduced by performing a boost and serves merely as a bookkeeping device.

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In order to prove that two operators in (1.2) are equal, it is sufficient to prove that they have the same eigenvalues. If they do, then there is guaranteed to exist a unitary transformation between the spaces on which the two operators act. A basis independent formulation of the BMN correspondence, therefore, is that the interacting string field theory hamiltonian $\frac{2}{\mu}P^-$ and the gauge theory operator H must have the same eigenvalues.

While this is the minimum that we are allowed to expect from the BMN correspondence, we can hope to do better. Light-cone string field theory, as formulated in [4]–[6], comes with a natural choice of basis: this string basis (of single and multiple strings) is neither the BMN basis (of single and multiple traces) nor the basis of eigenstates of the light-cone hamiltonian. But how do we identify the string basis in the gauge theory?

One guess for the string basis was made in [9, 13], where it was argued that matrix elements of P^- between 1- and 2-string states should be equated with the coefficient of the three-point function of the corresponding BMN operators, multiplied by the difference in conformal dimension between the incoming and outgoing states. This proposal appeared to be supported by the subsequent string field theory calculation done in [5] (see also [20]–[27]). It turns out, however, that the final step of the calculation in [5] suffered from a minus sign error (which we will correct below), which renders the alleged confirmation of this proposal invalid.

In this paper we propose a new, specific form for the transformation between the BMN basis and the string basis, valid to all orders in g_2. This basis transformation is trivial to write down, and has the pleasing feature that it does not depend on the conformal dimensions of the operators. In fact, our choice of transformation was already identified as a natural choice in [18], where it was shown that all computed amplitudes in gauge theory are reproduced via a relatively simple string bit formalism [15]–[17]. While most calculations in [18] were done in the BMN basis, it was pointed out that there exists a basis choice with the properties that (i) the inner product is diagonal, and (ii) the matrix elements of the supersymmetry generators Q are at most linear in g_2 (i.e. Q leads to only a single string splitting or joining). Here we will show that, when evaluated in this new basis, the matrix elements of the string bit hamiltonian, which via the results of [18] may be identified with the gauge theory operator on the right-hand side of (1.2), agree precisely with the corrected answer of [5] for the matrix elements of the continuum string field theory hamiltonian P^- appearing on the left-hand side!

The precise match between the three point functions means that, by combining the two formalisms, we can start filling in some important questions left open in [4] and [18]. A major technical obstacle in continuum light-cone string field theory is that higher order contact terms are needed for closure of the supersymmetry algebra, and that their value (at order g_2^2) affects the leading order shift in the eigenvalues of P^-. However, these contact terms are difficult to compute [19]. The supersymmetry algebra of the bit string theory, on the other hand, is known to all orders in g_2 but only to linear order in the fermions. It appears to be a fruitful strategy, therefore, to make use of the discretized theory to fix the order g_2^2 contact terms of the continuum theory, while the known non-linear fermionic form of the continuum interaction vertex may be of direct help in deriving the complete supersymmetry generators in the string bit formalism.

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2. Identification of the string basis in gauge theory

$\mathcal{N}=4$ gauge theory in the BMN limit comes with a natural choice of basis, which coincides with the natural string basis when $g_2=0$: an n string state corresponds to a product of n single trace BMN operators. We call this basis the BMN basis, denoted by $|\psi_n\rangle$. At non-zero g_2, the inner product (defined as the overlap as computed in the free gauge theory) becomes non-diagonal in this basis. The explicit form of the inner product is conveniently expressed in terms of the string bit language of [16, 18] as

$$\langle\psi_m|\psi_n\rangle_{g_2} = \left(e^{g_2\Sigma}\right)_{nm}, \qquad \Sigma = \frac{1}{J^2}\sum_{i<j}\Sigma_{ij}\,, \tag{2.1}$$

where Σ_{ij} is the operator which interchanges the string bits via the simple permutation (ij). As explained in [16, 18], when acting on a BMN state $|\psi\rangle$ with n strings, Σ effectuates a single string splitting or joining.

This meaning of Σ in the gauge theory language can be made concrete as follows. Consider a long BMN string in its ground state. We can write the corresponding operator as

$$\mathcal{O}_J(\gamma) = \mathrm{Tr}(Z^J) = \sum_{\substack{i_1\cdots i_J\\ \bar{i}_1\cdots\bar{i}_j}} Z_{i_1\bar{i}_1}\,Z_{i_2\bar{i}_2}\cdots Z_{i_J\bar{i}_J}\;\delta^{\bar{i}_1 i_{\gamma(1)}}\,\delta^{\bar{i}_2 i_{\gamma(2)}}\cdots\delta^{\bar{i}_J i_{\gamma(J)}} \tag{2.2}$$

with $\gamma = (12\ldots J)$ the cyclic permutation of J elements. The action of Σ_{J_1J}, which implements the simple permutation (J_1J), is now defined as

$$\Sigma_{J_1J}\,\mathcal{O}_J(\gamma) = \mathcal{O}_J(\gamma\circ(J_1J))\,. \tag{2.3}$$

Since $\gamma\circ(J_1J) = (1\ldots J_1\!-\!1\,J)(J_1\ldots J-1)$ we have that

$$\Sigma_{J_1J}\,\mathcal{O}_J(\gamma) = \mathrm{Tr}(Z^{J_1})\mathrm{Tr}(Z^{J-J_1})\,, \tag{2.4}$$

showing that the simple permutation Σ_{J_1J} indeed induces a single splitting of a single trace into a double trace operator. It is easy to generalize this result to other operators, to show that Σ can either split a string or join two strings.

The identification of (2.1) with the inner product of the free gauge theory was motivated in [18] and explicitly verified for string ground states to all order in g_2 and for two-impurity states to order g_2^2.

States with different number of strings are therefore no longer orthogonal relative to (2.1). In the string field theory basis $|\widetilde{\psi}_n\rangle$, on the other hand, the inner product should be diagonal for all g_2. The simplest basis transformation that achieves this goal is

$$|\widetilde{\psi}_n\rangle = \left(e^{-g_2\Sigma/2}\right)_{nm}|\psi_m\rangle\,. \tag{2.5}$$

This is not the most general diagonalization, however, since we still have the freedom to redefine the new basis $|\widetilde{\psi}_m\rangle$ via an arbitrary unitary transformation [12]. The above redefinition (2.5), however, has the attractive feature that it is purely combinatoric and does not depend on the dynamics of the gauge theory. Furthermore, as we will see shortly, it has the desirable property that the (linearized) supersymmetry generators and light-cone hamiltonian acquire a simple form in the new basis. We emphasize that the only way to check the proposal (2.5) for identifying the string field theory basis in the gauge theory is by comparing matrix elements of H calculated in the $|\widetilde{\psi}_n\rangle$ basis to those of $\frac{2}{\mu}P^-$ in light-cone string field theory. We show below that the proposal (2.5) passes this test.

In the following, we will study the consequences of this basis transformation for the specific class of two-impurity BMN states investigated in [7, 11, 9, 13, 12]. We will denote by $|1,p\rangle$ the normalized state corresponding to the single trace operator $\sum_l e^{2\pi i p l/J} \times \mathrm{Tr}(\phi Z^l \psi Z^{J-l})$, while $|2,k,y\rangle$ and $|2,y\rangle$ will denote the normalized states corresponding to the double trace operators $\sum_l e^{2\pi i k l/J_1}\mathrm{Tr}(\phi Z^l \psi Z^{J_1-l})\mathrm{Tr}(Z^{J-J_1})$ and $\mathrm{Tr}(\phi Z^{J_1})\mathrm{Tr}(\psi Z^{J-J_1})$ respectively, where $y = J_1/J$. The action of Σ on $|1,p\rangle$ reads

$$\Sigma\,|1,p\rangle = \sum_{k,y} C_{pky}\,|2,k,y\rangle \;+\; \sum_{y} C_{py}|2,y\rangle\,, \tag{2.6}$$

with

$$C_{pky} = \sqrt{\frac{1-y}{Jy}}\,\frac{\sin^2(\pi p y)}{\pi^2(p-k/y)^2}\,, \qquad C_{py} = -\frac{\sin^2(\pi p y)}{\sqrt{J}\pi^2 p^2}\,. \tag{2.7}$$

Via (2.5) we now introduce the corresponding two-impurity states in the string basis, which we will denote by $|\tilde{1},p,y\rangle$, $|\tilde{2},k,y\rangle$ and $|\tilde{2},y\rangle$, respectively. By construction, these form an orthonormal basis at finite g_2.

3. Interactions in the string basis

In this section we obtain the matrix elements of the right-hand side of (1.2) in the string basis proposed in the previous section. For this we will employ the string bit model of [18], but by virtue of the established correspondence with the gauge theory amplitudes of [11, 13], the following calculation can also be viewed as a direct calculation within the gauge theory.

It was shown in [18] that the linearized (in the fermions) interacting supercharges in the string bit model can be written in the string basis as

$$Q = Q_0 + \frac{g_2}{2}[\widehat{Q}_0, \Sigma]\,, \qquad \widehat{Q}_0 = Q_0^< - Q_0^>\,, \tag{3.1}$$

where $Q_0 = Q_0^< + Q_0^>$ is the free supercharge of the bit string theory and the superscripts indicate the projection onto the term with fermionic creation ($<$) or annihilation ($>$) operators only. These charges generate the interacting superalgebra of string theory in the plane wave background, modulo higher order terms in the fermions. Our interest is to compute the matrix elements of H between the bosonic two-impurity states in the string basis.

The supercharge (3.1) truncates at linear order in g_2. As indicated in fig 1, this truncation is expected from the BMN correspondence: matrix elements of the supercharges $Q = \mathrm{Tr}\,\theta[Z,\phi]$ in the gauge theory can lead (for connected diagrams, where Q has at least one contraction with either the “in” or “out” BMN state) to at most one single string splitting or joining [18]. For now, however, one may view (3.1) as a new starting point

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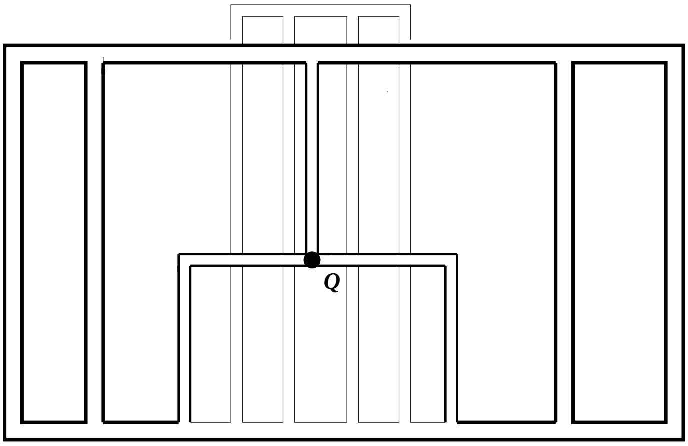

Figure 1: An insertion of the supercharge in the gauge theory will lead to a single splitting of the BMN string with which it has a double contraction.

of the string bit model; in the remainder we will re-establish its equivalence with the perturbative gauge theory, by showing that, at least for the special class of two-impurity states, it leads to the same order g_2^2 mass renormalization (shift in conformal dimension) as computed in [11, 13].

From (3.1) we thus deduce that the interacting hamiltonian truncates at order g_2^2:

$$H = H_0 + g_2 H_1 + g_2^2 H_2 \tag{3.2}$$

with[2]

$$H_0 = \{Q_0, Q_0\}\,, \qquad H_1 = \{Q_0, [\widehat{Q}_0, \Sigma]\}\,, \qquad H_2 = \frac{1}{4}\{[\widehat{Q}_0, \Sigma], [\widehat{Q}_0, \Sigma]\}\,. \tag{3.3}$$

From these expressions, it is straightforward to compute the matrix elements between the various two-impurity states. Using that $Q_0^>$ annihilates bosonic states, we find that

$$\langle\widetilde{\psi}_2|H_1|\widetilde{\psi}_1\rangle = \frac{1}{2}\langle\widetilde{\psi}_2|(H_0\Sigma + \Sigma H_0)|\widetilde{\psi}_1\rangle - 2\langle\widetilde{\psi}_2|Q_0^>\,\Sigma\,Q_0^<\,|\widetilde{\psi}_1\rangle\,. \tag{3.4}$$

Both matrix elements on the right-hand side have been computed in [18], with the result

$$\langle\tilde{2}, k, y|(H_0\Sigma + \Sigma H_0)|\tilde{1}, p\rangle \;=\; \lambda'\left(p^2 + \frac{k^2}{y^2}\right) C_{pky}\,, \tag{3.5}$$

$$\langle\tilde{2}, k, y|\,Q_0^>\Sigma\,Q_0^<\,|\tilde{1}, p\rangle \;=\; \frac{\lambda'}{2}\left(\frac{pk}{y}\right) C_{pky}\,, \tag{3.6}$$

where C_{pky} are the three point functions defined in eq. (2.7). Inserting the explicit expressions, we find

$$\langle\tilde{2}, k, y|H_1|\tilde{1}, p\rangle = \frac{\lambda'}{2}\sqrt{\frac{1-y}{Jy}}\frac{\sin^2(\pi p y)}{\pi^2}, \qquad \langle\tilde{2}, y|H_1|\tilde{1}, p\rangle = -\frac{\lambda'}{2}\frac{1}{\sqrt{J}}\frac{\sin^2(\pi p y)}{\pi^2}\,. \tag{3.7}$$

[2] The notation in these equations is somewhat symbolic: the left-hand side in each equation is equal to the projection onto the $\delta^{\dot{a}\dot{b}}$ component of the anti-commutator on the right-hand side. Furthermore, as stated above, this formula for the hamiltonian is valid only for computing matrix elements between bosonic states; for fermionic states, the non-linear fermionic corrections to (3.1) will become relevant.

In a similar way one can obtain the order g_2^2 matrix elements between single string states. We postpone this discussion to later, and turn now to the continuum string field theory.

4. Light-cone string field theory

We now investigate the matrix elements of the left-hand side of (1.2) in the continuum string theory, in order to compare with the gauge theory results of the previous section. In light-cone string field theory the cubic interaction is conveniently represented as a state in the three-string Hilbert space. If we restrict our attention to string states which have no fermionic excitations, this state can be expressed as

$$\frac{2}{\mu}|P_1^-\rangle = -\frac{y(1-y)}{2}\mathcal{P}|V\rangle\,. \tag{4.1}$$

Here $|V\rangle$ is a squeezed state in the 3-string Hilbert space,

$$|V\rangle = \exp\left[\frac{1}{2}\sum_{r,s=1}^{3}\sum_{m,n=-\infty}^{\infty} a^{I\dagger}_{m(r)}\overline{N}^{(rs)}_{mn}a^{I\dagger}_{n(s)}\delta_{IJ}\right]|0\rangle\,, \tag{4.2}$$

and the prefactor $\mathcal{P}$ is given by

$$\mathcal{P} = \sum_{r=1}^{3}\sum_{m=-\infty}^{\infty}\frac{\omega_{m(r)}}{\mu\alpha_{(r)}}a^{I\dagger}_{m(r)}a^{J}_{-m(r)}v_{IJ}\,, \tag{4.3}$$

where $v_{IJ} = \mathrm{diag}(1_4, -1_4)$, $\omega_{m(r)} = \sqrt{m^2+\mu^2\alpha_{(r)}^2}$, and $\alpha_{(r)} = \alpha' p^+_{(r)}$, with the convention that $\alpha_{(r)}$ is negative for incoming strings 1 and 2 and positive for the outgoing string 3. In order to write $\mathcal{P}$ in this form, we have employed a very useful factorization identity derived in [28, 29]. The sign error in the original version of [5] amounts, after tracing through some changes of basis, to replacing the a_{-m} by a_m in (4.3), making manifest the incorrect claim that the prefactor gives the difference of energy between incoming and outgoing states.[3]

Implicit formulas for the matrix elements of $\overline{N}$, valid for all λ', were presented in [4]. While explicit formulas for the leading terms in an expansion around $\lambda' = 0$ are known, it is very difficult to extract exact formulas for higher-order terms [30] (see [32] for the latest progress). At $\lambda' = 0$ the only nonzero Neumann matrices are

$$\begin{aligned}\overline{N}^{(13)}_{kp} &= \frac{(-1)^{k+p+1}}{\sqrt{y}}\frac{\sin(\pi p y)}{\pi(p-k/y)}\,,\\ \overline{N}^{(23)}_{kp} &= -\frac{1}{\sqrt{1-y}}\frac{\sin(\pi p(1-y))}{\pi(p-k/(1-y))}\,.\end{aligned} \tag{4.4}$$

Using the above formulas, it is straightforward to derive the leading $\mathcal{O}(\lambda')$ contribution to the matrix elements

$$\begin{aligned}\frac{2}{\mu}\langle\tilde{2},k,y|P_1^-|\tilde{1},p\rangle &= \frac{\lambda'}{2}(1-y)\frac{\sin^2(\pi p y)}{\pi^2}\,,\\ \frac{2}{\mu}\langle\tilde{2},y|P_1^-|\tilde{1},p\rangle &= -\frac{\lambda'}{2}\sqrt{y(1-y)}\frac{\sin^2(\pi p y)}{\pi^2}\,.\end{aligned} \tag{4.5}$$

[3]The error can also be understood as a missing factor of i in eqn (3.15) of [4], as pointed out in [29].

This result, which corrects the one originally reported in [5], is in precise agreement with (3.7) after taking into account the factor $\sqrt{Jy(1-y)}$ which arises because (3.7) is written in terms of unit normalized states while (4.5) is expressed in terms of continuum states satisfying $\langle i|j\rangle = p_i^+\delta(p_i^+ - p_j^+)$.

5. Contact terms and mass renormalization

Having established that the matrix elements of H evaluated in the $|\widetilde{\psi}_n\rangle$ basis agree at order g_2 with those of $\frac{2}{\mu}P^-$ in the natural string field theory basis, let us now revisit the issues of contact terms and the one-loop mass renormalization of the single-string state $|\tilde{1},p\rangle$.

A significant advantage of the $|\widetilde{\psi}\rangle$ basis is that the matrix elements of the supercharge (3.1) terminate at order g_2. Therefore the hamiltonian terminates at order g_2^2, with the term H_2 which comes from squaring the order g_2 term in the supercharge; there is no need for all of the higher-order contact terms which seem to plague continuum light-cone string field theory. Turning this observation around gives a definite prediction for string field theory: that the only contact term surviving in the large μ limit is the one which comes from squaring the cubic vertex in the dynamical supercharge.

The order g_2^2 matrix element between single string states has been computed in [18] (see equation (71)), with the result

$$\begin{aligned} g_2^2\langle\tilde{1},q|H_2|\tilde{1},p\rangle &= \frac{g_2^2}{4}\langle\tilde{1},q|\,[\,Q_0^>,\Sigma][\Sigma,Q_0^<\,]\,|\tilde{1},p\,\rangle \\ &= \frac{g_2^2\lambda'}{4}\sum_i\left(\frac{k^2}{y^2}-\frac{1}{2}(p^2+q^2)\right)C_{p\,i}C_{qi} \;\equiv \frac{1}{4}\frac{g_2^2\lambda'}{4\pi^2}B_{pq}. \end{aligned} \tag{5.1}$$

Here the sum runs over intermediate 2-string states i of both types: $|\tilde{2},k,y\rangle$ and $|\tilde{2},y\rangle$ (and includes an integral over y). The explicit form of B_{pq} is as given in [11, 13].

To obtain the order g_2^2 mass renormalization of the state $|\tilde{1},p\rangle$, we should add the matrix element (5.1) to the iterated H_1 interaction:[4]

$$\begin{aligned} \sum_i \frac{|\langle\,\tilde{1},p\,|\,H_1|\,i\,\rangle|^2}{E_p-E_i} &= \frac{1}{4}g_2^2\lambda'\sum_i\frac{(p-k/y)^4C_{pi}^2}{p^2-k^2/y^2} \\ &= \frac{1}{4}g_2^2\lambda'\sum_i\left(p^2-\frac{k^2}{y^2}\right)C_{pi}C_{-pi} \\ &= -\frac{1}{4}\frac{g_2^2\lambda'}{4\pi^2}B_{p,-p}\,. \end{aligned} \tag{5.2}$$

An additional subtlety in the calculation is that the states $|\tilde{1},p\rangle$ and $|\tilde{1},-p\rangle$ are degenerate at lowest order; we should check therefore whether we need to use degenerate perturbation theory. It is easy to see that

$$\sum_i \frac{\langle\,\tilde{1},p\,|\,H_1|\,i\,\rangle\langle\,i\,|\,H_1|\,\tilde{1},-p\rangle}{E_p-E_i} = -\frac{1}{4}\frac{g_2^2\lambda'}{4\pi^2}B_{p,-p} \tag{5.3}$$

[4] Here in the second step we use that $C_{-pky} = \frac{(p-k/y)^2}{(p+k/y)^2}\,C_{pky}$.

gives the same result as (5.2). The sum of the contact term H_2 and the iterated H_1 interaction is diagonal in the $\{|\tilde{1}, p\rangle, |\tilde{1}, -p\rangle\}$ basis, signalling that the degeneracy remains unbroken to this order.[5]

Putting everything together, we find that the order g_2^2 contribution to the eigenvalue is

$$\frac{1}{4}\frac{g_2^2\lambda'}{4\pi^2}(B_{pp} - B_{p,-p}) = \frac{g_2^2\lambda'}{4\pi^2}\left(\frac{1}{12} + \frac{35}{32\pi^2 p^2}\right). \tag{5.4}$$

This agrees precisely with the shift in anomalous dimension of the conformal eigen-operators, as reported in [11, 13].

6. Conclusion

We have proposed an explicit form (2.5) of the basis transformation that relates single and multi-trace BMN operators in the gauge theory to single and multi-string states of the dual string field theory in the plane wave background to all orders in g_2. This basis transformation is natural from the point of view of the bit string theory of [18]: besides the fact that it diagonalizes the inner product, it has the property that the supersymmetry generators in the new basis truncate at linear order in g_2.

Our most encouraging result, however, is that the 3-point function in this basis precisely matches with the 3-string amplitude of the continuum string field theory [5]. In itself this match does not yet prove anything, because one can always find two bases that would lead to the same 3-point function. One also needs control over the order g_2^2 contact interactions before one can honestly compare the shift in the conformal dimensions in the gauge theory with the mass renormalization in the string theory [19]. However, we have more information than just the 3-point function: because the supersymmetry charge of the bit string model is linear in g_2, via the closure of the supersymmetry algebra we have a principle that uniquely determines the order g_2^2 contact interaction. It also tells us that any higher order contact terms are absent.

It would clearly be of interest to give a precise construction of the continuum limit of the bit string theory by taking the large J limit while keeping λ' fixed. The correspondence found in this paper is an encouraging indication that this continuum limit will coincide with the continuum light-cone string theory.

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[5] In fact, supersymmetry requires these two states to be exactly degenerate [13]. The fact that we find degeneracy at this order is consistent with our observation that no additional contact terms are required for closure of the supersymmetry algebra.

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